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DETERMINATION OF STRESS INTENSITY FACTORS AND CRACK-OPENING STRESSES FROM JUMPS IN CRACK-EDGE DISPLACEMENTS

V. N. Maksimenko

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In accordance with the superposition principle, stress-intensity factors (SIFs) — which control crack growth — can be calculated through the distribution of the nominal (crack-opening) stresses acting at the site of a crack in an undamaged structure. In actual structures, these stresses may differ appreciably from the values predicted theoretically. Various theoretical-experimental methods are used to determine the nominal stresses (cutting out layers, drilling holes, cutting notches [1-5]) and the SIF from the strain fields in the region ahead of a crack tip (extensometry, recording of the opening of the crack near its tip by means of sensors or computer analysis of visual images, laser-assisted speckle methods, photoelastic and holographic methods [4-10], etc.). However, in most cases these methods are flawed by several deficiencies: the time consumed in different stages of machining; limitations of computational procedures in regard to specific configurations; a certain arbitrariness and lack of rigor attending the use of the methods.

Using integral representations of solutions of problems concerning the elastic equilibrium of anisotropic plates weakened by a curvilinear slit (crack), here we proposed a method which makes it possible to use experimental jumps in crack-edge displacement found for several points to calculate two different stress-intensity factors and the distribution of the nominal stresses on the line of the crack in complex sectional structural elements made of metallic and composite materials. The efficiency and accuracy of the proposed approach is evaluated by mathematically modeling several problems of practical importance and comparing the results with experimental data.

1. We will take a loaded structure and isolate a plane element representing a plate made of an elastic, rectilinearly anisotropic (specifically, isotropic) material. The element occupies the finite region D in the plane xOy, and it contains a system of holes and cracks. The geometry of the structural element is shown in Fig. 1.

We will assume that the slit (crack) L passes completely through the element and that its edges are free of external forces. The structure is loaded in such a way that a plane stress state is realized in it in the absence of slit L.

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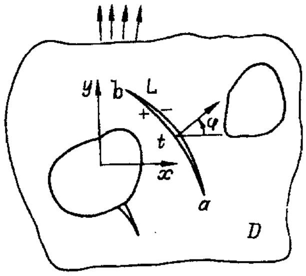

Fig. 1

We need to determine the SIF on the basis of known jumps in the displacements $G(t) = [(u + iv)^{+} - (u + iv)^{-}] = g_1(t) + ig_2(t)$ at the edges of the slit (Fig. 1).

The stress in the plate in the region of the defect can be expressed in terms of two analytic functions:

$$(\sigma_x, \tau_{xy}, \sigma_y) = 2\,\mathrm{Re}\left\{\sum_{\nu=1}^{2} (\mu_\nu^2, -\mu_\nu, 1)\, \Phi_\nu(z_\nu)\right\}, \tag{1.1}$$

where $z_\nu = x + iy$; μ_ν are the roots of the corresponding characterized equation (with positive imaginary parts [11]).

Following [12], we write the functions $\Phi_\nu(z_\nu)$ in the form

$$\Phi_\nu(z_\nu) = \sum_{j=0}^{1} \Phi_{\nu j}(z_\nu); \tag{1.2}$$

$$\Phi_{\nu 1}(z_\nu) = \frac{1}{2\pi i}\int_L \frac{\omega_\nu(\tau)\, d\tau_\nu}{\tau_\nu - z_\nu}; \tag{1.3}$$

$$a(t)\,\omega_1(t) + b(t)\,\overline{\omega_1(t)} + \omega_2(t) = 0, \tag{1.4}$$

$$a(t) = a_0 \frac{M_1(t)}{M_2(t)}, \quad b(t) = b_0 \frac{\overline{M_1(t)}}{M_2(t)},$$

$$a_0 = (\mu_1 - \bar{\mu}_2)(\mu_2 - \bar{\mu}_2)^{-1}, \quad b_0 = (\bar{\mu}_1 - \bar{\mu}_2)(\mu_2 - \bar{\mu}_2)^{-1}.$$

Here, $\Phi_{\nu 0}(z_\nu)$ determines the main stress state, while $\Phi_{\nu 1}(z_\nu)$ is the perturbed stress state which arises in the presence of the slit L; $\phi = \phi(t)$ is the angle between the x-axis and the normal n to the left edge of slit L at point t; ds is an element of the arc of the contour of L; $\omega_1(t)$ belongs to the class of functions which are not bounded at the ends a, b of slit L [13], i.e., this function can be represented in the form

$$\omega_1(t) = \frac{\Omega^*(t)}{\sqrt{(t-a)(t-b)}} \quad (t \in L), \tag{1.5}$$

where $\Omega^*(t)$ is a function of the class H on L in the neighborhood of the ends of the slit [13]; $\sqrt{(t-a)(t-b)}$ is any branch which changes continuously on L.

With allowance for (1.3) and the relations

$$(u, v) = 2\,\mathrm{Re}\left\{\sum_{\nu=1}^{2} (p_\nu, q_\nu)\, \varphi_\nu(z_\nu)\right\}, \quad \varphi_\nu'(z_\nu) = \Phi_\nu(z_\nu)$$

we can find the jump in displacements w(t) on L has the form

$$G(t) = \sum_{\nu=1}^{2}\left\{(p_\nu + iq_\nu)\int_{a_j}^{t} \omega_\nu(\tau)\, d\tau_\nu + (\bar{p}_\nu + i\bar{q}_\nu)\int_{a_j}^{t} \overline{\omega_\nu(\tau)}\, d\bar{\tau}_\nu\right\}. \tag{1.6}$$

We can easily use (1.6) to establish the physical significance of the functions $\omega_1(t)$. Differentiating (1.6), we obtain

$$\frac{dw}{dt} = \sum_{\nu=1}^{2} \left\{(p_\nu + iq_\nu) M_\nu(t) \omega_\nu(t) + (\bar{p}_\nu + i\bar{q}_\nu) M_\nu(t) \overline{\omega_\nu(t)}\right\}. \tag{1.7}$$

Thus, it follows from (1.4), (1.7) that $\omega_\nu(t)$ can be expressed directly through the derivatives of the displacement jumps on L:

$$\omega_1(t) = \frac{W(t)[A(t) - a(t)] - \overline{W(t)}[B(t) - b(t)]}{|A(t) - a(t)|^2 + |B(t) - b(t)|^2}, \tag{1.8}$$

$$\omega_2(t) = -a(t)\omega_1(t) - b(t)\overline{\omega_1(t)},$$

$$W(t) = \left(\bar{p}_2 \frac{dg_2(t)}{ds} - \bar{q}_2 \frac{dg_1(t)}{ds}\right) [(\bar{p}_2 q_2 - p_2 \bar{q}_2) M_2(t)]^{-1},$$

$$A(t) = A_0 M_1(t)/M_2(t), \quad B(t) = B_0 \overline{M_1(t)}/M_2(t),$$

$$A_0 = (\bar{p}_2 q_1 - p_1 \bar{q}_2)(\bar{p}_2 q_2 - p_2 \bar{q}_2)^{-1}, \quad B_0 = (\bar{p}_2 \bar{q}_1 - \bar{p}_1 \bar{q}_2)(\bar{p}_2 q_2 - p_2 \bar{q}_2)^{-1}.$$

For the given case of an internal crack, we can use (1.6) and conditions corresponding to continuity of the displacements at the tips of the cracks to obtain the following additional condition for $\omega_1(t)$:

$$\int_L \omega_1(\tau)\, d\tau_1 = 0. \tag{1.9}$$

Let the equation of the contour of L be described by the relation $t = \tau(\eta)$ ($a = \tau(-1)$, $b = \tau(1)$), where η is a dimensionless real-valued parameter. Then, with allowance for (1.5), we can represent the function $\omega_1(t)$ in the form

$$\omega_1(t) = \omega_1[\tau(\eta)] = \chi(\eta) = \chi^0(\eta)(1 - \eta^2)^{-1/2}. \tag{1.10}$$

Having determined $\omega_1(t)$ (and, thus, $\chi^0(\pm 1)$) through the derivatives of the displacement jumps on L by means of Eq. (1.8), we use (1.1-1.2) and (1.9) to obtain asymptotic formulas for the stresses in the neighborhood $c = \tau(\mp 1)$ of the ends of the slit L:

$$\lim_{r \to 0} \sqrt{2r}\,(\sigma_x, \tau_{xy}, \sigma_y) = \mathrm{Re}\left\{\left(\pm \left.\frac{ds}{d\eta}\right|_{\eta=\pm 1}\right)^{1/2} \sum_{\nu=1}^{2} (\mu_\nu^2, -\mu_\nu, 1)\, C_\nu(\vartheta)\right\}, \tag{1.11}$$

$$C_\nu(\vartheta) = \Omega_\nu [M_\nu(c)(\cos\vartheta + \mu_\nu \sin\vartheta)^{-1}]^{1/2},$$

$$\Omega_1 = \chi^0(\pm 1), \quad \Omega_2 = -a(c)\Omega_1 - b(c)\overline{\Omega}_1,$$

as well as the SIFs for normal rupture and shear $K_{1,2}$ [14].

2. We will represent the potentials $\Phi_\nu(z_\nu)$ for the cracked plate in the form

$$\Phi_\nu(z_\nu) = \sum_{j=0}^{2} \Phi_{\nu j}^*(z_\nu), \tag{2.1}$$

where $\Phi_{\nu 1}^*(z_\nu) \equiv \Phi_{\nu 1}(z_\nu)$ (see (1.3)), while the potentials $\Phi_\nu^*(z_\nu) = \sum_{j=1}^{2} \Phi_{\nu j}^*(z_\nu)$ correspond to the case when the external loads applied to the body are equal to zero everywhere except for the edges of the slit L. Then obviously the potentials $\Phi_{\nu 0}^*(z_\nu)$ will determine the nominal stresses in the undamaged plate from the action of the same system of external forces.

Having inserted (2.1) into the boundary conditions for L [12], we have

$$a(t)\Phi_1^\pm(t_1) + b(t)\overline{\Phi_1^\pm(t_1)} + \Phi_2^\pm(t_2) = 0. \tag{2.2}$$

With allowance for (1.3-1.4) and the properties of the potentials $\Phi_{\nu 0}^*(z_\nu)$, we find from (2.1-2.2) that

$$X_n^0(t) + \bar{\mu}_2 Y_n^0(t) = (\bar{\mu}_2 - \mu_2) M_2(t) \sum_{j=1}^{2} \{a(t)\Phi_{1j}^*(t_1) + b(t)\overline{\Phi_{1j}^*(t_1)} + \Phi_{2j}^*(t_2)\} \tag{2.3}$$

($X_n^0(t)ds$, $Y_n^0(t)ds$ are projections of the nominal stresses acting on a curved element ds of the contour of L in a plate without a crack).

We will examine three special cases of the problem formulated above.

A. Let the damaged element be an infinite plate with a crack L. Then $\Phi_{\nu 2}^*(z_\nu) \equiv 0$, and after allowing for (1.3) and completing certain transformations we obtain the following from (2.3) to explicitly express the nominal stresses on the line of the defect in terms of the function $\omega_1(t)$:

$$X_n^0(t) + \bar{\mu}_2 Y_n^0(t) = C(t)\left\{\int_L \frac{\omega_1(\tau)\,d\tau_1}{\tau_1 - t_1} + \int_L [K_1(t,\tau)\,\omega_1(\tau) + K_2(t,\tau)\,\overline{\omega_1(\tau)}]\,ds\right\}, \tag{2.4}$$

$$C(t) = M_1(t)(\bar{\mu}_2 - \bar{\mu}_1)(\pi i)^{-1},$$

$$K_1(t,\tau)\,ds = \frac{1}{2}\left\{d\ln\frac{\bar{\tau}_2 - \bar{t}_2}{\tau_1 - t_1} + \frac{\overline{b(\tau)} - \overline{b(t)}}{\overline{b(t)}(\bar{\tau}_2 - \bar{t}_2)}\,d\bar{\tau}_2\right\},$$

$$K_2(t,\tau)\,ds = \frac{1}{2}\left\{d\ln\frac{\bar{\tau}_2 - \bar{t}_2}{\bar{\tau}_1 - \bar{t}_1} + \frac{\overline{a(\tau)} - \overline{a(t)}}{\overline{b(t)}(\bar{\tau}_2 - \bar{t}_2)}\,d\bar{\tau}_2\right\}.$$

If the crack is located along a straight line, then Eq. (2.4) is simplified considerably: $K_1(t, \tau) = K_2(t, \tau) \equiv 0$. For example, for a crack $L = \{|x| < a,\ y = 0\}$, the nominal stresses $\sigma_y^0(x, 0)|$, $\tau_{xy}^0(x, 0)$ acting at the site of the crack L in the infinite plate are given by the formula

$$\tau_{xy}^0(x, 0) + \bar{\mu}_2\sigma_y^0(x, 0) = \frac{\bar{\mu}_2 - \bar{\mu}_1}{\pi i}\int_{-a}^{a}\frac{\omega_1(\tau)\,d\tau}{\tau - x}. \tag{2.5}$$

Limiting ourselves to examination of a type-I crack and proceeding analogously to [12] by taking the limit in (1.3), (2.5) for the case of an isotropic medium, we arrive at the relations obtained in [4].

B. Let a plate loaded by a system of external forces occupy the half-plane $D = \{x > 0\}$ and be weakened by a crack L originating from or located close to the straight edge x = 0 of the plate. Then in accordance with [12]

$$\Phi_{\nu 2}^*(z_\nu) = \frac{1}{2\pi i}\int_L \left\{\frac{l_\nu s_\nu \overline{\omega_1(\tau)}\,d\bar{\tau}_1}{s_\nu z_\nu - \bar{\tau}_1} + \frac{n_\nu m_\nu \overline{\omega_2(\tau)}\,d\bar{\tau}_2}{m_\nu z_\nu - \bar{\tau}_2}\right\}, \tag{2.6}$$

$$l_\nu = \frac{\mu_{3-\nu} - \bar{\mu}_1}{\mu_\nu - \mu_{3-\nu}},\quad n_\nu = \frac{\mu_{3-\nu} - \bar{\mu}_2}{\mu_\nu - \mu_{3-\nu}},\quad s_\nu = \frac{\bar{\mu}_1}{\mu_\nu},\quad m_\nu = \frac{\bar{\mu}_2}{\mu_\nu}\quad (\nu = 1, 2).$$

The thus-constructed potentials $\Phi_\nu^*(z_\nu)$ automatically satisfy zero boundary conditions for the stresses on the edge of the half-plane x = 0 and at infinity. Allowing for (2.1) and (2.6), we use the same method and (2.3) to find that the nominal stresses on the line of the crack are determined by a relation of the form (2.3). The kernels $K_j(t, \tau)$ are more complex in form for this case and are omitted here.

C. Let the region D occupied by the plate be an infinite plane xOy outside an elliptical hole $\Lambda = \{(x/a)^2 + (y/a)^2 = 1\}$, and let the crack L (internal or edge) be located next to Λ. The closed analytic representations of the potentials $\Phi_{\nu 2}^*(z_\nu)$ in [15], expressed through the function $\omega_1(t)$, automatically satify the boundary conditions on the edge of the elliptical hole Λ which is free of external forces and decay at infinity. With allowance for the results in [15], we also find that, as previously, the nominal stresses on the line L in the undamaged plate are determined through the function $\omega_1(t)$ by means of Eqs. (2.4) (due to its awkwardness, the explicit form of $K_j(t, \tau)$ is not presented here.

In accordance with St. Venant principle, Eqs. (2.4-2.5) (case A) will satisfactorily approximate the distribution of the nominal stresses at the site of the crack for finite plates as well if the perturbed stress state which develops due to the presence of the slit L is localized and does not propagate to the boundaries of the plate. If the defect is located near the straight edge of the plate, the elliptical hole, or the boundary of the body (being an arc of an ellipse) and if the effect of the other boundaries can be ignored, then Eqs. (2.4) can be used to determine the nominal stresses for cases B and C, respectively.

3. We will assign the jumps of the crack-edge displacements along the x-axis $g_1^p = (u^+ - u^-)^p$ and y-axis $g_2^q = (v^+ - v^-)^q$ at arbitrary points $t_p = \tau(\beta_p) \in L$ $(p = \overline{1, N_1})$ and $t_{N_1+q} = \tau(\beta_{N_1+q}) \in L$ $(q = \overline{1, N_2})$, respectively†.

We represent the function G(t) as an approximating series in Chebyshev functions of the second kind $U_k(\beta) = \sin(k \arccos \beta)$ [16]:

$$G(t) = G(\tau(\beta)) = G^*(\beta) = \sum_{k=1}^{M} A_k U_k(\beta). \tag{3.1}$$

Here, $A_k = a_k + ib_k$ are unknown complex constants.

We will use the least squares method to determine A_k. To do this, we examine the functional

$$S(a_1, b_1, a_2, b_2, \ldots, a_M, b_M) = \sum_{p=1}^{N_1} \left(\sum_{k=1}^{M} a_k U_k(\beta_p) - g_1^p \right)^2 + \sum_{q=1}^{N_2} \left(\sum_{k=1}^{M} b_k U_k(\beta_{N_1+q}) - g_2^q \right)^2.$$

Then the necessary condition of the extremum yields a system of 2M linear algebraic equations to find a_j, b_j $(j = \overline{1, M})$:

$$\frac{\partial S}{\partial a_j} = 2 \sum_{p=1}^{N_1} \left(\sum_{k=1}^{M} a_k U_k(\beta_p) - g_1^p \right) U_j(\beta_p) = 0, \tag{3.2}$$

$$\frac{\partial S}{\partial b_j} = 2 \sum_{q=1}^{N_2} \left(\sum_{k=1}^{M} b_k U_k(\beta_{N_1+q}) - g_2^q \right) U_j(\beta_{N_1+q}) = 0.$$

Having determined A_k from (3.2) and using (3.1) and the relation from [16]

$$\frac{dU_k(\beta)}{d\beta} = -\frac{kT_k(\beta)}{\sqrt{1-\beta^2}}, \quad T_k(\beta) = \cos(k \arccos \beta) \quad (k = 1, 2, \ldots)$$

we write an expression for the derivative of the displacement jumps:

$$\frac{dG}{d\beta} = \frac{g_1^0(\beta) + ig_2^0(\beta)}{\sqrt{1-\beta^2}} = -\frac{1}{\sqrt{1-\beta^2}} \sum_{k=1}^{M} kA_k T_k(\beta). \tag{3.3}$$

In accordance with (1.8), (1.10), and (3.3), we have

$$\chi^0(\beta) = \frac{W_0(\beta)[A(t) - a(t)] - \overline{W_0(\beta)}[B(t) - b(t)]}{|A(t) - a(t)|^2 + |B(t) - b(t)|^2} \left(\frac{ds}{d\beta}\right)^{-1}, \tag{3.4}$$

$$W_0(\beta) = [\bar{p}_2 g_2^0(\beta) - \bar{q}_2 g_1^0(\beta)] [(\bar{p}_2 q_2 - p_2 \bar{q}_2) M_2(t)]^{-1}.$$

Knowing $\chi^0(\beta)$, we use Eqs. (1.11) to obtain the SIF and asymptotic formulas for the stresses in the neighborhood of the crack tips.

With allowance for substitution of variables (1.10), Eq. (2.4) can be represented in the form

†In particular, the points $t_j = \tau(\beta_j)$ $(j = \overline{1, N_1 + N_2})$ may coincide.

$$X_n^0(t) + \bar{\mu}_2 Y_n^0(t) = \int_{-1}^{1} \left\{ \left[\frac{F(\beta, \eta)}{\eta - \beta} + k_1(\beta, \eta) \right] \chi^0(\eta) + k_2(\beta, \eta) \overline{\chi^0(\eta)} \right\} \frac{d\eta}{\sqrt{1-\eta^2}}, \quad (3.5)$$

$$F(\beta, \eta) = \frac{(\eta - \beta)\dot{\tau}_1(\eta)}{\tau_1(\eta) - \tau_1(\beta)} C(\beta), \quad C(\beta) = C(\tau(\beta)),$$

$$k_1(\beta, \eta) = \frac{ds}{d\eta} K_1[\tau(\beta), \tau(\eta)] C(\beta), \quad \dot{\tau}(\beta) = \frac{d\tau}{d\beta},$$

$$k_2(\beta, \eta) = \frac{ds}{d\eta} K_2[\tau(\beta), \tau(\eta)] C(\beta),$$

where $F(\beta, \eta)$, $k_j(\beta, \eta)$ $(j = 1, 2)$ are continuous functions.

To calculate the integral in (3.5), we use the formula in [17]

$$\int_{-1}^{1} \frac{\chi^0(\eta)}{\sqrt{1-\eta^2}} K(\beta, \eta)\, d\eta = \frac{\pi}{R} \sum_{m=1}^{R} \chi^0(\eta_m) K(\beta, \eta_m),$$

$$\eta_m = \cos\frac{2m-1}{2R}\pi \quad (m = \overline{1, R}),$$

which is valid for regular integrals with any β and for singular integrals with

$$\beta = \beta_r = \cos\frac{\pi}{R} r \quad (r = \overline{1, R-1})$$

(the number of nodes R is an arbitrary natural number). We then arrive at a quadratic formula to determine the nominal forces at Chebyshev nodes $t_r = \tau(\beta_r)$ $(r = \overline{1, R-1})$:

$$X_n^0(t_r) + \bar{\mu}_2 Y_n^0(t_r) = \frac{\pi}{R} \sum_{m=1}^{R} \{k_1^*(\beta_r, \eta_m) \chi^0(\eta_m) + k_2(\beta_r, \eta_m) \overline{\chi^0(\eta_m)}\},$$

$$k_1^*(\beta, \eta) = \frac{F(\beta, \eta)}{\eta - \beta} + k_1(\beta, \eta).$$

4. From a practical viewpoint, the proposed method of calculating the SIFs $K_{1,2}$ and the nominal stresses on the line of the defect is useful only if acceptable results can be obtained on the basis of measurement of the crack-edge displacements at a small number of points removed from the tip. We performed a numerical experiment to study the error of the proposed method in relation to the number and location of the points where the displacement of the edges is determined. Data for the isotropic material was obtained by taking the limit in the anisotropy parameters in the numerical solution.

As examples of use of the method, we chose the following types of problems for isotropic plates with type-I cracks: a) a central crack in a tensioned strip of finite width, $a/W = 0.7$; b) an edge crack in a tensioned half-plane; c) an internal crack in a tensioned half-plane, $d/a = 0.1$; d) a hole with an edge crack in a tensioned plane, $a/r = 0.1$; e) a crack next to a hole in a tensioned plane, $a/r = 1$, $d/a = 0.1$; f) an infinite plate with a crack whose edges are loaded by a pair of symmetrically applied concentrated forces, $b/a = 0.5$ (Fig. 2a-f). The solutions of these problems either have closed expressions [14] or are obtained by means of an application package employing the SIU method [12, 15]. In calculating the SIF from formulas (1.11), (3.1-3.4), we used values of crack-opening at 1-5 nodes located on the contour of the crack at points with the coordinates $\beta_k^{(1)} = x_k/a = 0;\ 0.2;\ 0.4;\ 0.6;\ 0.8$ and $\beta_k^{(2)} = 0.1;\ 0.3;\ 0.5;\ 0.7;\ 0.9$ $(k = \overline{1, 5})$.

Table 1 shows the errors of the calculation of the SIF for normal rupture K_1 at the left and right tips of a crack δ $(\mp a)$ in relation to the number of points p $(p = \overline{1, 5})$ with coordinates $\beta_k^{(j)}$ $(k = \overline{1, p})$, where the opening of the crack was given (measured). The solutions for the families of nodes $\{\beta_k^{(1)}\}$, $\{\beta_k^{(2)}\}$ $(k = \overline{1, p})$ are given above and below the lines, respectively. Here and below, the number of terms of approximating series M in Eq. (3.1) was taken equal to p.

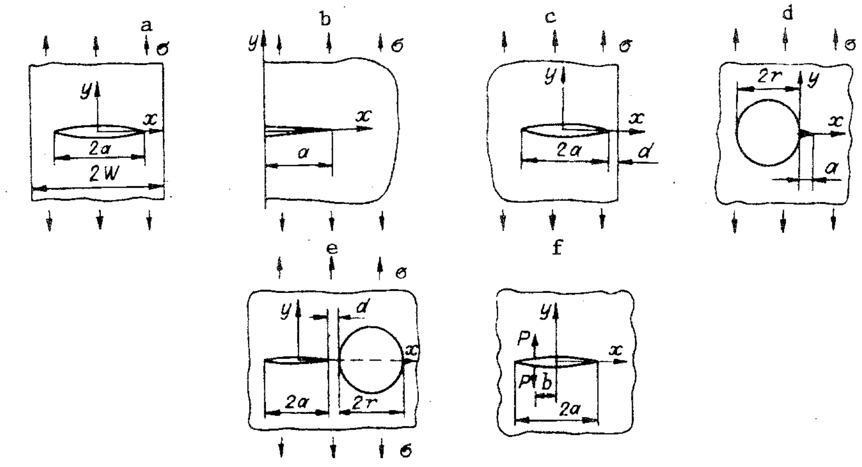

Fig. 2

TABLE 1

Type of problem	Number of points, p									
	1		2		3		4		5	
	δ(−a)	δ(a)	δ(−a)	δ(a)	δ(−a)	δ(a)	δ(−a)	δ(a)	δ(−a)	δ(a)
a	−3,4 / −3,4	−3,4 / −3,4	−4,0 / −4,9	−2,8 / −2,1	0,2 / −0,2	0,1 / −0,1	0,1 / −0,4	0,1 / −0,1	−2,2 / −1,4	−0,1 / −0,1
b	—	29,0 / 23,1	—	25,8 / −17,8	—	10,4 / 6,8	—	−1,4 / −1,3	—	−0,3 / −0,1
c	11,4 / 13,3	−23,3 / −22,0	−8,9 / −11,8	−9,3 / −7,8	8,3 / 12,4	−1,4 / −0,5	−7,0 / −10,5	1,6 / 1,9	1,3 / −44,2	2,0 / 1,3
d	—	34,4 / 28,2	—	−23,9 / −16,2	—	11,1 / 6,9	—	−2,8 / −1,8	—	0,2 / 0,1
e	12,8 / 15,2	−36,2 / −34,9	−12,6 / −17,5	−21,9 / −19,8	16,1 / 26,3	−11,1 / −9,0	−29,5 / −53,8	−3,8 / −2,3	56,7 / 121,6	−0,3 / 0,3
f	−24,0 / −32,8	128,1 / 101,7	54,6 / 36,1	−107 / −67,2	136,9 / 107,4	57,1 / 27,1	223,8 / 181,9	−17,4 / −5,7	316,4 / 260,1	2,3 / 0,4

TABLE 2

No. of nodes	N	β_k										δ(−a)	δ(a)
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9		
2	1	+		+								−12,6	−21,9
	2			+		+						−24,0	−17,6
	3					+		+				−46,5	−12,2
	4							+		+		−92,4	−5,7
	5		+		+							−17,5	−19,8
	6				+		+					−33,5	−15,0
	7						+		+			−65,1	−9,0
	8								+		+	−132,1	−2,3
4	1			+	+							−4,1	−8,6
	2					+	+					−3,3	−6,7
	3							+	+			−2,0	−3,9
	4									+	+	−0,6	−0,6
	5		+		+							−3,7	−8,9
	6				+		+					−3,3	−7,2
	7						+		+			−2,3	−4,5
	8								+		+	−0,8	−1,0

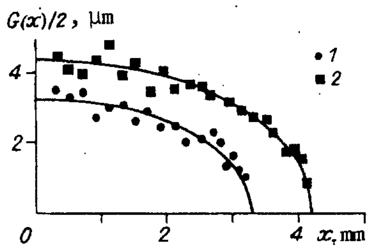

Fig. 3

For comparison, Table 2 shows the error δ (±a) of the calculations for problem e with eight variants (N = $\overline{1, 8}$) of crack-opening measurement at two and four nodes positioned symmetrically relative to the center of the crack; the positive coordinates of the chosen nodes are designated by the plus sign.

An analysis of the calculated results presented in Tables 1 and 2 shows that, for problems a-e, the proposed method determines the SIF from crack-opening values at two or three points quite far from the tip with an error of 1-7%. The best results are obtained as the observation points approach the tip and increase in number. If the loads are applied to the edges of the defect and are local in character (such as in the case of a crack on a loaded fastener hole), the accuracy of the method declines somewhat (see Table 1, problem f).

Let us make use of the experimental data reported in [4] on the opening of the edges of a central crack in a strip of steel S45C tensioned by forces σ = 100 MPa (Fig. 2a, 2W = 35 mm, sheet thickness 2 mm). Points 1 and 2 in Fig. 3 show these results for the half-length of the crack when a = 3.3 and 4.2 mm. In accordance with the proposed method, we used this empirical data and (3.1) to determine crack-opening G(t) with M = 1 (line). We also calculated the SIF K_1 and the miminal stresses on the line of the crack. The error of the determination of the SIF relative to the theoretical solution [14] is 1.6 and 5.4% when a = 3.3 and 4.2 mm. The error of the nominal stresses calculated from Eq. (2.5) for σ = 100 MPa is 3.9 and 9.0%, respectively.

The results of the numerical modeling and the comparison with the experimental data show the high degree of accuracy and reliability of the proposed method. Among its advantages are quickness, flexibility (different shapes of structural elements and different shapes, locations, and types of cracks, isotropic or anisotropic materials, etc.), the possibility of reliably determining the stresses at the site of a crack in an undamaged structure, and the possibility of determining the first and second stress-intensity factors on the basis of crack-opening data at two or three points away from the tip. The simplicity and efficiency of the proposed method makes it possible to recommend it for practical use in evaluating the strength and predicting the safe life of structure.

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PROBE DIAGNOSIS OF A FLOW OF PARTICLES DESORBED FROM THE SURFACE OF A SOLID BY A LOW-DENSITY PLASMA JET

V. Z. Korn and V. A. Shuvalov

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The dynamic interaction of bodies with a flow of low-density gas is characterized by a variety of processes and phenomena occurring at the phase boundary. If the energy of the incident particles is greater than approximately 5 eV, processes involving energy and momentum transfer are accompanied by the dispersal of surface contaminants and layers of adsorbed gases, the desorption of particles from the surface, etc. Monitoring and study of these processes are very important from phenomenological and practical viewpoints to establish a balance between transfers of momentum, mass, and energy at the phase boundary. The parameters of mass flows dispersed by inflowing particles are usually measured by the gravimetric method [1, 2]. However, this method does not distinguish between the fraction of particles desorbed from the surface due to the dispersal of adsorbed layers and coatings by the incoming flow and the fraction of the loss due to erosion of the material of the surface [3]. Such a distinction is important for describing mass transfer in gas-surface systems and momentum and energy transfer at phase boundaries.

In the present article, we describe the methodology and results of an experimental study of the parameters of particles desorbed from a surface as a result from its bombardment by an inflowing low-density plasma. It is shown that the proposed method makes it possible to determine the fraction of particles adsorbed on the surface of the solid and evaluate their parameters.

1. A body placed in a high-velocity flow of a low-density plasma is subjected not only to incoming neutral and charged particles accelerated in the near-electrode layer, but also to particles desorbed from the surface by its intensive bombardment. Within a certain range of surface potentials, the particle desorption stimulated by the incident flow results in an increase in the total momentum transferred to the surface of the body. In a detailed examination of the balance of forces acting on a body in a low-density plasma flow, the momentum imparted to the body can be determined in the form

$$F_\Sigma^{(1)}(V) = F_i(V) + F_e(V) + \Delta F$$

for a surface free of adsorbent and

$$F_\Sigma^{(2)}(V) = F_i(V) + F_e(V) + \Delta F + F_d$$

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